Formelsammlung DSP

DGl's:

cooking recipe: (i) homogenious solution: $\alpha(D)\, y_h(n) = 0$

$\rightarrow \alpha(z_i^{-1}) = 0$

↳ single root: $y_h = c_i z_i^n$

multiple %: $y_h = (c_1 + c_2 n + \dots + c_m n^{m-1})\, z_i^n$

(ii) particular solution:

*) Exp. signal $x(n) = A \cdot q^n$

$$\alpha(q^{-1}) \neq 0 \Rightarrow y_p(n) = A\,\frac{\beta(q^{-1})}{\alpha(q^{-1})}\, q^n$$

$$\alpha(q^{-1}) = 0\ (\#m) \Rightarrow y_p(n) = A\,\frac{\beta(q^{-1})}{\alpha^{(m)}(q^{-1})}\,(-qn)^m\, q^n$$

$$\alpha^{(m)}(D) = \frac{\partial^m \alpha(D)}{\partial D}$$

*) Power signal $x(n) = A n^q$

↳ $y_p(n) = B_0 n^q + B_1 n^{q-1} + \dots + B_q$

(determine B_i: $\alpha(D)\, y_p(n) = \beta(D)\, x(n)$)

(iii) total solution: $y(n) = y_h(n) + y_p(n)$

(iv) determine parameters c_i to satisfy initial cond.

Zero-state: $y_{zs}(n) = y(n)|_{\text{initial values}=0}$

Zero-input: $y_{zi}(n) = y(n)|_{x(n)=0}$

(attention: if initial values $\neq 0$

↳ system non-linear:

$$y(n) = y_{zs_1} + y_{zs_2} + y_{zi}$$

*) $$(1 - r e^{j\theta} z^{-1})(1 - r e^{-j\theta} z^{-1}) = 1 - 2r\cos(\theta)\, z^{-1} + r^2 z^{-2}$$

*) $$|1 \pm e^{-j\omega N}| = 2\overset{\cos}{\sin}\left(\omega \frac{N}{2}\right)$$

linear phase → conj. compl. zeros

Stability

Geometric Series

$$\sum_{n=M}^{N-1} a^n = \begin{cases} \frac{a^M - a^N}{1-a} & a \neq 1 \\ N - M & a = 1 \end{cases}$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1$$

$$\sum_{n=0}^{\infty} n \cdot a^n = \frac{a}{(1-a)^2} \qquad |a| < 1$$

Convolution

$$(x*h)(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

[N.M "·"

(N-1)(M-1) "+"

$(x*h)(n) = (h*x)(n)$

$(x*h_1)*h_2 = x*(h_1*h_2)$

$x*h_1 + x*h_2 = x*(h_1+h_2)$

Z-transform:

Def.: $X(z) = \sum_n x(n)\, z^{-n} \leftrightarrow x(n) = \frac{1}{2\pi j}\oint_C X(z)\, z^{n-1}\, dz$

Properties:

*) $x(-n) \rightarrow X(\frac{1}{z})$

*) $x^*(n) \rightarrow X^*(z^*)$

*) $\sum c_i x_i(n) \rightarrow \sum c_i X_i(z)$

*) $x(n-k) \rightarrow z^{-k} X(z)$

*) $a^n x(n) \rightarrow X(\frac{z}{a}),\ a \neq 0$

*) $\mathrm{Re}\{x(n)\} \rightarrow \frac{1}{2}[X(z) + X^*(z^*)]$

*) $n \cdot x(n) \rightarrow -z \frac{d}{dz} X(z)$

*) $\sum_n n \cdot x(n) = -z \frac{d}{dz} X(z)|_{z=1}$

*) $(x_1 * x_2)(n) \rightarrow X_1(z) \cdot X_2(z)$

*) $x(0) = \begin{cases} X(\infty) & \text{if } x(n) \text{ is causal} \\ X(0) & \text{if } x(n) \text{ is anticausal} \end{cases}$

$x(n)$	$\rightarrow X(z)$	ROC
$\delta(n)$	1	$\mathbb{C}$
$\delta(n-k)$	z^{-k}	$z \neq 0$ if $k>0$, $z \neq \infty$ if $k<0$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$n \cdot a^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u(-n-1)$	%	$\lvert z\rvert < \lvert a\rvert$
$c_k(n)\, a^n u(n)$	$\frac{1}{(1-az^{-1})^k}$	$\lvert z\rvert > \lvert a\rvert$
$-c_k(n)\, a^n u(-n-k)$	%	$\lvert z\rvert < \lvert a\rvert$
$a^{\lvert n\rvert}$	$\frac{1-a^2}{(1-az^{-1})(1-az)}$	$\lvert a\rvert < \lvert z\rvert < \frac{1}{\lvert a\rvert}$
$a^n \cos(\omega_0 n)\, u(n)$	$\frac{1 - a\cos(\omega_0) z^{-1}}{1 - 2a\cos\omega_0 z^{-1} + a^2 z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
$a^n \sin(\omega_0 n)\, u(n)$	$\frac{a \sin\omega_0 z^{-1}}{1 - 2a\cos\omega_0 z^{-1} + a^2 z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
$\frac{1}{n!}\, u(n)$	$e^{z^{-1}}$	$z \neq 0$

with $c_1(n) = 1$, $c_2(n) = n+1$, $c_3(n) = \frac{(n+1)(n+2)}{2}$

$$c_k(n) = \sum_{i=0}^{n} c_{k-1}(i)$$

Fourier series:

$$x(n) = \sum_{k=0}^{N-1} c_k\, e^{j 2\pi \frac{k}{N} n}$$

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n)\, e^{-j 2\pi \frac{k}{N} n}$$

$$c_{k+N} = c_k$$

Sampling theorem:

$$W_2 = \{ 1, -1 \}$$

$$W_3 = \{ 1 \mid -\frac{1}{2} - \frac{\sqrt{3}}{2} j \mid -\frac{1}{2} + \frac{\sqrt{3}}{2} j \}$$

$$W_4 = \{ 1 \mid -j \mid -1 \mid j \}$$

$$W_6 = \{ 1 \mid \frac{1}{2} - \frac{\sqrt{3}}{2} j \mid -\frac{1}{2} - \frac{\sqrt{3}}{2} j \mid -1 \mid -\frac{1}{2} + \frac{\sqrt{3}}{2} j \mid \frac{1}{2} + \frac{\sqrt{3}}{2} j \}$$

$$W_8 = \{ 1 \mid \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} j \mid -j \mid -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} j \mid -1 \mid -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} j \mid j \mid \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} j \}$$